THEORY OF THE ARCH.

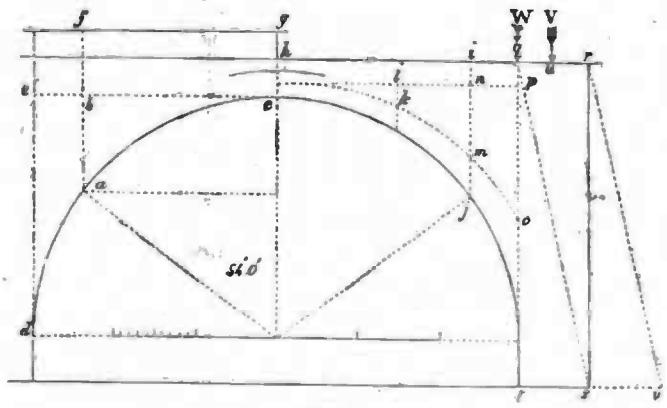

Fig. 1.

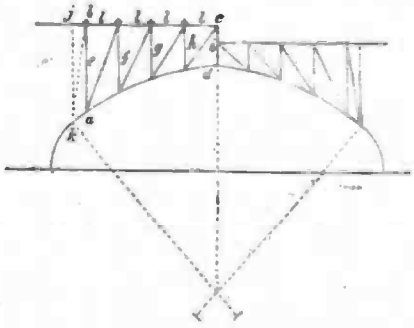

Fig. 5.

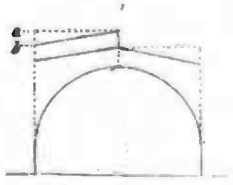

Fig. 2.

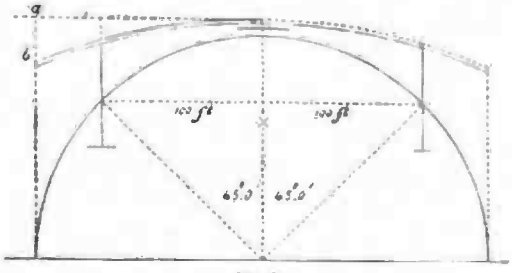

Fig. 3.

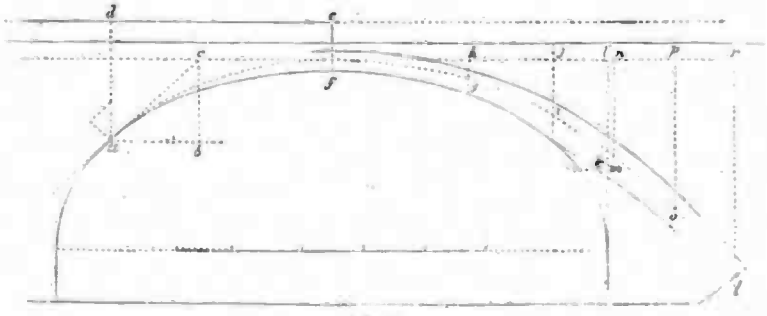

Fig. 4.

THEORY OF THE ARCH APPLIED.*

By means of the annexed table, the points of fracture, the horizontal stress, points in the curve of equilibrium, &c., of circular and elliptical arches, may be determined with considerable accuracy and facility. Its use in comparing bridges of different dimensions, and for other purposes, will be shewn by a few examples.

Table to aid in calculating the Horizontal Stress, &c., of Circular and Elliptical Arches, with various loads.

Circ. Ellip.	Ver-sine R H	Sine R R	Sine² R² R²	Sine³ R³ R³	Sine⁴ R⁴ R⁴	A R³ RH	D R R	M R³ R³H
5°	·00381	·08716	·00760	·00066	·00006	·00011	·02171	·000003
10°	·01519	·17365	·03015	·00524	·00093	·00088	·04334	·00004
15°	·03407	·25882	·06699	·01734	·00449	·00292	·06446	·00019
20°	·06031	·34202	·11698	·04001	·01368	·00679	·08498	·00058
25°	·09369	·42262	·17861	·07548	·03190	·01294	·10464	·00135
30°	·13397	·50000	·25000	·12500	·06250	·02169	·12326	·00267
35°	·18085	·57358	·32899	·18870	·10823	·03322	·14067	·00465
40°	·23396	·64279	·41318	·26558	·17071	·04752	·15669	·00745
45°	·29289	·70711	·50000	·35355	·25000	·06441	·17235	·01102
46°	·30534	·71934	·51745	·37223	·26775	·06807	·17388	·01184
47°	·31800	·73135	·53488	·39119	·28609	·07181	·17651	·01268
48°	·33087	·74314	·55226	·41041	·30500	·07564	·17907	·01354
49°	·34394	·75471	·56959	·42987	·32443	·07954	·18157	·01444
50°	·35721	·76604	·58682	·44953	·34436	·08351	·18399	·01537
51°	·37068	·77715	·60396	·46936	·36476	·08755	·18635	·01632
52°	·38434	·78801	·62096	·48932	·38559	·09165	·18863	·01729
53°	·39818	·79864	·63782	·50938	·40681	·09581	·19084	·01828
54°	·41221	·80902	·65451	·52951	·42838	·10001	·19298	·01930
55°	·42642	·81915	·67101	·54966	·45026	·10426	·19504	·02033
56°	·44081	·82904	·68730	·56980	·47238	·10855	·19700	·02138
57°	·45536	·83867	·70336	·58989	·49473	·11287	·19883	·02245
58°	·47008	·84805	·71919	·60990	·51723	·11720	·20078	·02353
59°	·48496	·85717	·73474	·62979	·53984	·12156	·20254	·02462
60°	·50000	·86603	·75000	·64952	·56250	·12592	·20423	·02572
90°	1·0000	1·0000	1·0000	1·0000	1·0000	·21460	·22337	·04794

*For an explanation of the particular theory adopted and applied, see an article under the same title in the fourth volume of THE BUILDER, page 618, to which this may be considered an appendix.

The second column of the table contains versed sines of arcs of a circle; the third, fourth, fifth, and sixth columns, the first, second, third, and fourth powers of sines. The seventh column, headed A, contains the areas of the curvilinear triangles ($a\ b\ e$), ($d\ e\ e$), &c., Fig. 1.; the eighth, marked D, the distances of their centres of gravity from their vertical sides ($a\ b$), ($d\ e$), &c.; the ninth, marked M, their moments;—the whole corresponding to arcs of the number of degrees figured in the first column, the radius being unity. The values of the quantities corresponding to any other radius or semi-span, R, are to be obtained by multiplying the dimensions of the table by the powers of R, shewn in the first line under the heading of it. For elliptical arches, the quantities in the second line are to be used, in which R is the semi-span and H the rise. The values of A, D, and M were calculated by the following formulæ—

$$A = \text{sine} \times \text{ver-sine} + \text{sine} - \text{arc}$$

$$D = \frac{\text{sine} - \frac{\text{ver-sine}^2}{2} - \frac{\text{ver-sine}^3}{3}}{A}$$

$$M = A \text{ sine} - \frac{\text{ver-sine}^3}{2} + \frac{\text{ver-sine}^3}{3}$$

To find the area proportional to the horizontal (S) of a semi-circular arch loaded as represented by Fig. 1. To the moment of ($f\ b\ e\ g$), add the moment of ($a\ b\ e$), and divide the sum by the versed sine ($e\ k$), added to half the depth of the key ($\frac{K}{2}$). If the